1. Let f : R $\to$ R be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)} =$ [AIEEE-2010]

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

2. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ [AIEEE-2011]

(1) equals $-\sqrt{2}$ (2) equals $\frac{1}{\sqrt{2}}$

(3) does not exist (4) equals $\sqrt{2}$

3. Let f : R $\to$ [0, ∞) be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}} = 0$ Then $\lim_{x\to 5} f(x)$ equal - [AIEEE-2011]

(1) 3 (2) 0 (3) 1 (4) 2

4. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to: [JEE(Main)-2014]

(1) $\frac{\pi}{2}$ (2) 1 (3) $-\pi$ (4) π

5. If $\lim_{x\to 2}\frac{\tan(x-2)\{x^2+(k-2)x-2k\}}{x^2-4x+4} = 5$ then k is equal to [JEE(Main)-2014]

(1) 3 (2) 1 (3) 0 (4) 2

6. Let p = $\lim_{x\to 0+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to: [JEE(Main)-2016]

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

7. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals: [JEE(Main)-2017]

(1) $\frac{1}{4}$ (2) $\frac{1}{24}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

8. For each t $\in$ R, let [t] be the greatest integer less than or equal to t. Then

$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....\left[\frac{15}{x}\right]\right)$ [JEE(Main)-2018]

(1) does not exist (in R) (2) is equal to 0. (3) is equal to 15. (4) is equal to 120.

9. $\lim_{y\to 0}\dfrac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$ [JEE(Main)-2019]

(1) exists and equals $\dfrac{1}{4\sqrt{2}}$ (2) does not exist

(3) exists and equals $\dfrac{1}{2\sqrt{2}}$ (4) exists and equals $\dfrac{1}{2\sqrt{2}(\sqrt{2}+1)}$

10. For each $x \in R$, let [x] be the greatest integer less than or equal to x. Then

$\lim_{x\to 0^-}\dfrac{x\left([x]+|x|\right)\sin[x]}{|x|}$ [JEE(Main)-2019]

(1) $-\sin 1$ (2) $\sin 1$ (3) 1 (4) 0

11. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then,

$\lim_{x\to 1^+}\dfrac{(1-|x|+\sin|1-x|)\sin\left(\frac{\pi}{2}[1-x]\right)}{|1-x|[1-x]}$ [JEE(Main)-2019]

(1) equals 0 (2) does not exist

(3) equals -1 (4) equals 1

12. Let [x] denote the greatest integer less than or equal to x.

Then: $\lim_{x\to 0}\dfrac{\tan\left(\pi\sin^2 x\right)+\left(|x|-\sin\left(x[x]\right)\right)^2}{x^2}$ [JEE(Main)-2019]

(1) does not exist (2) equals π (3) equals 0 (4) equals $\pi + 1$

13. $\lim_{x\to 0}\dfrac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to: [JEE(Main)-2019]

(1) 1 (2) 4 (3) 0 (4) 2

14. $\lim_{x\to \pi/4}\dfrac{\cot^3 x-\tan x}{\cos\left(x+\pi/4\right)}$ is: [JEE(Main)-2019]

(1) $4\sqrt{2}$ (2) 8 (3) 4 (4) $8\sqrt{2}$

15. $\lim_{x\to 1^-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to: [JEE(Main)-2019]

(1) $\sqrt{\pi}$ (2) $\sqrt{\dfrac{\pi}{2}}$ (3) $\sqrt{\dfrac{2}{\pi}}$ (4) $\dfrac{1}{\sqrt{2\pi}}$

16. $\lim_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals [JEE(Main)-2019]

(1) $2\sqrt{2}$ (2) 4 (3) $4\sqrt{2}$ (4) $\sqrt{2}$

17. If $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is: [JEE(Main)-2019]

(1) $\frac{3}{2}$ (2) $\frac{4}{3}$ (3) $\frac{3}{8}$ (4) $\frac{8}{3}$

18. If $\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5$, then a + b is equal to: [JEE(Main)-2019]

(1) – 7 (2) 5 (3) – 4 (4) 1

19. $\lim_{x\to 0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$ is: [JEE(Main)-2019]

(1) 3 (2) 2 (3) 1 (4) 6

20. Let f(x) = 5 – |x – 2| and g(x) = |x + 1|, x ∈ R. If f(x) attains maximum value at α and g(x) attains minimum value at β, then $\lim_{x\to -\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$ is equal to: [JEE(Main)-2019]

(1) – 3/2 (2) 3/2 (3) 1/2 (4) – 1/2

21. $\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to ______. [JEE(Main)-2020]

22. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to [JEE(Main)-2020]

(1) e^2 (2) e (3) $\frac{1}{e}$ (4) $\frac{1}{e^2}$

23. If $\lim_{x\to 1}\frac{x+x^2+x^3+...+x^n-n}{x-1}$ = 820, (n ∈ N) then the value of n is equal to ______.

[JEE(Main)-2020]

24. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}}$ is equal to: [JEE(Main)-2020]

(1) e (2) 2 (3) e^2 (4) 1

25. Let [t] denote the greatest integer ≤ t. If for some $\lambda \in R - \{0, 1\}$, $\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|$ = L, then L is equal to : [JEE(Main)-2020]

(1) $\frac{1}{2}$ (2) 0 (3) 2 (4) 1

26. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=2^{-k}$, then the value of k is ______.

[JEE(Main)-2020]

27. $\lim_{x\to a}\frac{(a+2x)^{\frac{1}{3}}-(3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}}-(4x)^{\frac{1}{3}}}$ $(a \neq 0)$ is equal to: [JEE(Main)-2020]

(1) $\left(\frac{2}{9}\right)^{\frac{4}{3}}$ (2) $\left(\frac{2}{3}\right)^{\frac{4}{3}}$ (3) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$

28. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x\to\alpha^+}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ is equal to [JEE(Main)-2020]

(1) $\frac{3}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

29. $\lim_{x\to 0}\frac{x\left(e^{(\sqrt{1+x^2+x^4}-1)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$ [JEE(Main)-2020]

(1) does not exist. (2) is equal to $\sqrt{e}$ (3) is equal to 1. (4) is equal to 0.

30. $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+..........\frac{1}{n}}{n^2}\right)^n$ is equal to: [JEE(Main)–2021]

(1) $\frac{1}{2}$ (2) $\frac{1}{e}$ (3) 1 (4) 0

31. The value of $\lim_{h\to 0}\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cosh-\sinh)}\right\}$ is: [JEE(Main)–2021]

(1) $\frac{3}{4}$ (2) $\frac{2}{\sqrt{3}}$ (3) $\frac{4}{3}$ (4) $\frac{2}{3}$

32. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a + b + c is equal to ______. [JEE(Main)–2021]

33. The value of $\lim_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2).\sin^{-1}(x-[x]^2)}{x-x^3}$, where [x] denotes the greatest integer $\leq$ x is:

[JEE(Main)–2021]

(1) π (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

34. If $\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of $(6L+1)$ is : [JEE(Main)–2021]

(1) 1/6 (2) 1/2 (3) 6 (4) 2

35. $\lim_{x\to\frac{\pi}{2}}\left(\tan^2 x\left((2\sin^2 x+3\sin x+4)^{\frac{1}{2}}-(\sin^2 x+6\sin x+2)^{\frac{1}{2}}\right)\right)$ is equal to: [JEE(Main)–2021]

(1) 1/12 (2) −1/18 (3) −1/2 (4) −1/6

36. If $\lim_{x\to 1}\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of (a − b) is equal to. [JEE(Main)–2022]

37. The value of $\lim_{x\to 1}\frac{\left(x^2-1\right)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to : [JEE(Main)–2022]

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{3}$ (3) $\frac{\pi^2}{2}$ (4) π^2

38. If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha+\beta)$ is equal to: [JEE(Main)–2022]

(1) 4 (2) −8 (3) −4 (4) 8

39. $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to: [JEE(Main)–2022]

(1) 14 (2) 7 (3) $14\sqrt{2}$ (4) $7\sqrt{2}$

40. If $\lim_{x\to 0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$, where $\alpha,\beta,\gamma\in R$ then which of the following is NOT correct ?

[JEE(Main)–2022]

(1) $\alpha^2+\beta^2+\gamma^2=6$ (2) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(3) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$ (4) $\alpha^2-\beta^2+\gamma^2=4$

ANSWER KEY

1.	(1)	2.	(3)	3.	(1)	4.	(4)	5.	(1)	6.	(4)	7.	(3)
8.	(4)	9.	(1)	10.	(1)	11.	(1)	12.	(1)	13.	(1)	14.	(2)
15.	(3)	16.	(3)	17.	(4)	18.	(1)	19.	(2)	20.	(3)	21.	36
22.	(4)	23.	40	24.	(3)	25.	(3)	26.	8	27.	(4)	28.	(1)
29.	(3)	30.	(3)	31.	(3)	32.	4	33.	(4)	34.	(4)	35.	(1)
36.	11	37.	(4)	38.	(3)	39.	(1)	40.	(3)				

1. If $\lim\limits_{x\to 0}\left[1+x\ell n(1+b^2)\right]^{\frac{1}{x}}$ = 2 b$\sin^2\theta$, b > 0 and $\theta \in (-\pi, \pi]$, then the value of θ is:

[JEE(Advanced)-2012]

(A) $\pm\dfrac{\pi}{4}$ (B) $\pm\dfrac{\pi}{3}$ (C) $\pm\dfrac{\pi}{6}$ (D) $\pm\dfrac{\pi}{2}$

2. If $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x+1}{x+1}-ax-b\right) = 4$, then: [JEE(Advanced)-2012]

(A) a = 1, b = 4 (B) a = 1, b = –4 (C) a = 2, b = –3 (D) a = 2, b = 3

3. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$ where a > – 1. then $\lim\limits_{a\to 0^+}\alpha(a)$ and $\lim\limits_{a\to 0^+}\beta(a)$ are: [JEE(Advanced)-2012]

(A) $-\dfrac{5}{2}$ and 1 (B) $-\dfrac{1}{2}$ and –1 (C) $-\dfrac{7}{2}$ and 2 (D) $-\dfrac{9}{2}$ and 3

4. Let L = $\lim\limits_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\dfrac{x^2}{4}}{x^4}$, a > 0. If L is finite, then: [JEE(Advanced)-2009]

(A) a = 2 (B) a = 1 (C) L = $\dfrac{1}{64}$ (D) L = $\dfrac{1}{32}$

5. The largest value of the non-negative integer a for which $\lim\limits_{x\to 1}\left\{\dfrac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \dfrac{1}{4}$ is:

[JEE(Advanced)-2014]

6. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{\alpha x-\sin x} = 1$. Then $6(\alpha+\beta)$ equals: [JEE(Advanced)-2016]

7. Let f(x) = $\dfrac{1-x(1+|1-x|)}{|1-x|}\cos\left(\dfrac{1}{1-x}\right)$ for $x \neq 1$, Then: [JEE(Advanced)-2017]

(A) $\lim\limits_{x\to 1^+} f(x)$ does not exist (B) $\lim\limits_{x\to 1^-} f(x)$ does not exist

(C) $\lim\limits_{x\to 1^+} f(x) = 0$ (D) $\lim\limits_{x\to 1^-} f(x) = 0$

8. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. We say that f has [JEE(Advanced)-2019]

PROPERTY 1 if $\lim\limits_{h \to 0} \frac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite, and

PROPERTY 2 if $\lim\limits_{h \to 0} \frac{f(h)-f(0)}{h^2}$ exists and is finite.

Then which of the following options is/are correct?

(A) $f(x) = x^{2/3}$ has PROPERTY 1

(B) $f(x) = \sin x$ has PROPERTY 2

(C) $f(x) = x|x|$ has PROPERTY 2

(D) $f(x) = |x|$ has PROPERTY 1

9. Let e denote the base of the natural logarithm. The value of the real number a for which the right-hand limit $\lim\limits_{x \to 0^+} \frac{(1-x)^{\frac{1}{x}} - e^{-1}}{x^a}$ is equal to nonzero real number, is ______.

[JEE(Advanced)-2020]

10. The value of the limit

$$\lim_{x \to \frac{\pi}{2}} \frac{4\sqrt{2}(\sin 3x + \sin x)}{\left(2\sin 2x \sin\frac{3x}{2} + \cos\frac{5x}{2}\right) - \left(\sqrt{2} + \sqrt{2}\cos 2x + \cos\frac{3x}{2}\right)}$$ is______ [JEE(Advanced)-2020]

11. For any positive integer n, let $S_n : (0,\infty) \rightarrow \mathbb{R}$ be defined by [JEE(Advanced)-2021]

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right)$$

where for any $x \in \mathbb{R}, \cot^{-1}(x) \in (0,\pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE ?

(A) $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x > 0$

(B) $\lim\limits_{n\to\infty} \cot\left(S_n(x)\right) = x$, for all $x > 0$

(C) The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0,\infty)$

(D) $\tan\left(S_n(x)\right) \le \frac{1}{2}$, for all $n \ge 1$ and $x > 0$

12. Let α be a positive real number. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: (\alpha, \infty) \to \mathbb{R}$ be the functions defined by

$$f(x) = \sin\left(\frac{\pi x}{12}\right) \quad \text{and} \quad g(x) = \frac{2\log_e(\sqrt{x}-\sqrt{\alpha})}{\log_e(e^{\sqrt{x}}-e^{\sqrt{\alpha}})}.$$

Then the value of $\lim\limits_{x\to\alpha^+} f(g(x))$ is__________. [JEE(Advanced)-2022]

13. If $\beta = \lim\limits_{x\to 0} \dfrac{e^{x^3} - \left(1-x^3\right)^{\frac{1}{3}} + \left(\left(1-x^2\right)^{\frac{1}{2}} - 1\right)\sin x}{x\sin^2 x}$, Then the value of 6β is __________.

[JEE(Advanced)-2022]

ANSWER KEY

1.	(D)	2.	(B)	3.	(B)	4.	(AC)	5.	0	6.	7	7.	(AD)
8.	(AD)	9.	1	10.	8	11.	(AB)	12.	0.50	13.	5		