



Mode of Heat Transfer





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Mode of Heat Transfer

- The transfer of energy from a body at higher temperature to a body at lower temperature is called heat transfer.

There are three types of mode of heat transfer:

Conduction:

- The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.

Convection:

- The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant in fluids (liquids & gases).

Radiation:

- Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the intervening medium.

Definition

The transfer of energy from a body at higher temperature to a body at lower temperature is called heat transfer.

Key Points

- ♦ Convection
- ♦ Conduction
- ♦ Radiation

Conductions	Convections	Radiation
• Heat transfer due to temperature difference.	• Heat transfer due to density difference.	• Heat transfer without any medium
• Due to free electron or vibration motion of molecules	• Actual motion of particle	• In the form of rays
• Heat transfer in solid Body	• Heat transfer in fluids (Liquid + gas)	• All



• Slow process	• Slow process	• Fast process (3×10^8 m/sec.)
• Irregular path	• Irregular path	• Straight line (like light-radiation)
		• It makes shadow also

1. Convection:

- In this mode of heat transfer medium is required and for heat transfer medium molecules is transferred from one place to another place.
- Convection is possible by earth gravity.
- In liquid and gases heat is transferred by convection.

Type of convection: -

Natural convection: Natural convection is possible by density difference and gravity.

- Direction of natural convection is from bottom to top [upward].
- In natural convection hot particles are move in upward direction and cold particles move in downward direction.
- If gravity effect is zero then natural convection is not possible like in space, artificial satellite, free falling lift and at centre of earth.

Examples: Heating of water, formation of monsoon, change in environment are several example of natural convection.

- If liquid and gases are heated from the top then heat is transferred by **conduction not** convection.
- Hg is liquid but it is heated by conduction and not by convection.

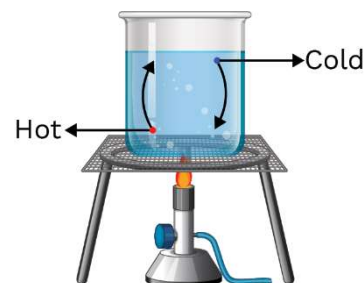
Definition

The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant in fluids (liquids & gases).



Concept Reminder

Natural convection is not possible without gravity.





Force convection: If for heat transfer, medium molecule is transferred by external force, then this mode of convection is called force convection.

- Direction of force convection is in external force direction.

Phenomena Based on convection:

1. Formation of trade winds
2. Ventilation
3. Monsoons
4. To regulate temperature in the human body.
5. Land and sea breezes

Important Points:

1. Natural convection takes place from bottom to top due to gravity while forced convection can occur in any direction.
2. In case of natural convection, convection currents move warm air upwards and cool air downward. That is why heating is done from base, while cooling from the top.
3. Natural convection is not possible in a gravity free region such as a freely falling lift or an orbiting satellite.
4. Natural convection plays an important role in ventilation in changing climate and weather and in forming land and sea breezes and trade winds.
5. The forced convection of blood in human body by a pump (heart) helps in keeping the temperature of body constant.
6. Convection takes place in fluids (gases and liquids)

2. Conduction:

- In this mode of heat transfer, medium is required but for heat transfer medium molecules is not displaced but it will transfer the energy to neighbour particles.

Key Points

- ♦ Natural convection
- ♦ Force convection

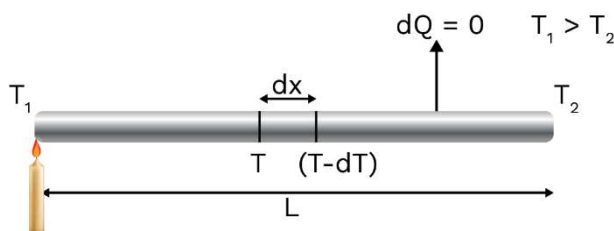


Concept Reminder

In conduction, heat transfer is due to movement of electron from one end to another as well as due to vibration of lattice of material.



- In solid and Hg heat is transferred by conduction.



- We heat a rod from its one end as shown above in figure. When we continue heat up its one end then slowly other parts also become hot as time passes. Firstly, all parts of rod absorb heat and after some time no heat is absorbed by any part. This is explained under two states.

(a) Variable state:

- If a rod is heated at one end then molecule in each cross-section will absorb some heat and will transfer some heat to next cross-section. Then temperature of each cross-section is increase and this process is cause variable state. (in variable state temperature is different at different points)

(b) Stable or steady state:

- In each cross-section if all the heat given to molecules of cross-section is completely transferred then it will be stable state.
- In each cross-section temperature is constant but different.

Rate of heat flow: In given above diagram, in steady state the heat flow through the rod is constant. The rate of heat flow $\frac{dQ}{dt}$ depends on-

- a. Cross section area (A) of rod.
- b. Rate of change of temperature with distance

$$\left(-\frac{dT}{dx} \right) = \text{temperature gradient}$$



Concept Reminder

- ◆ In variable state, temperature of each cross-section vary with time.
- ◆ In steady state, temperature of each cross-section remains constant with time.



Key Points

- ◆ Rate of heat flow
- ◆ Coefficient of conductivity



$$\therefore \frac{dQ}{dt} \propto A$$

$$\frac{dQ}{dt} \propto \left(-\frac{dT}{dx} \right)$$

On combining the relation

$$\frac{dQ}{dt} \propto A \left(-\frac{dT}{dx} \right)$$

$$\frac{dQ}{dt} = -KA \left(\frac{dT}{dx} \right) \quad \text{where}$$

$K \rightarrow$ coefficient of conductivity.

If we take the whole length of rod, then -

$$\frac{dQ}{dt} = -KA \left(\frac{T_2 - T_1}{L} \right)$$

$$\frac{dQ}{dt} = \frac{KA}{L} (T_1 - T_2) \rightarrow \text{Rate of heat flow}$$

Where $K \rightarrow$ coefficient of conductivity.

$A \rightarrow$ cross section area (Always perpendicular to heat flow direction)

$L \rightarrow$ Length of rod (along heat flow direction)

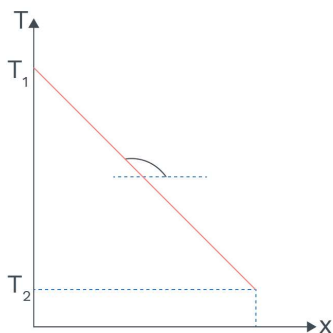
$T_1 \rightarrow$ High temperature

$T_2 \rightarrow$ Low temperature

Temperature gradient:

- The rate of change in temperature with respect to distance is called temperature gradient.

$$\text{Temperature gradient} = -\frac{dT}{dx}$$



Concept Reminder

Even at same temperature, the metal will feel colder than wood because of thermal conductivity of metal is greater than wood.



Concept Reminder

Value of K differentiate between good conductor and bad conductor of heat.

Rack your Brain



Which of the following rods, (given radius r and length l) each made of the same material and whose ends are maintained at the same temperature will conduct most heat?

- (1) $r = r_0, l = l_0$
- (2) $r = 2r_0, l = l_0$
- (3) $r = r_0, l = 2l_0$
- (4) $r = 2r_0, l = 2l_0$



$$\tan \theta = \text{constant} = -\frac{dT}{dx} = \text{temperature gradient}$$

- Thermal conductivity depends on nature of material it does not depend on shape and size of material.
- Order of thermal conductivity $\text{Ag} > \text{Cu} > \text{Au} > \text{Al}$
- When a substance is **good conductor** of heat then value of **K is high** and for **perfect conductors** [Ideal] $[K = \infty]$
- When a substance is **bad conductor** of heat then value of **K is low** and for **perfect insulator** [Ideal] $[K = 0]$
- Air and ice are bad conductor of heat.
- Human body is a bad conductor of heat, but it is a good conductor of current.
- For cooking utensils, thermal conductivity is high and specific heat is low.

**Concept Reminder**

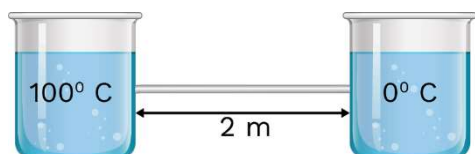
It is possible that a material is bad conductor of electricity but good conductor of heat.

Application of thermal conduction:

1. We feel warm in woollen clothes and fur coat.
2. A new quilt is warmer than old one.
3. Birds often swell their feathers in winter.
4. Ice is covered in gunny bags to prevent melting of ice.
5. In winter, the iron chairs appear to be colder than the wooden chairs.

Ex. Find the rate of heat flow in the following figure-

Given: $K = 200 \frac{\text{W}}{\text{m}^\circ\text{C}}, \quad \ell = 2\text{m}, \quad A = 10^{-2}\text{m}^2$



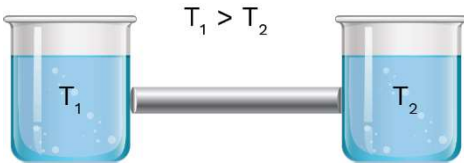
Sol.

$$i_T = \frac{Q}{t} = \frac{KA\Delta T}{\ell} = \frac{200 \times 10^{-2} \times 100}{2} = 100 \text{ J / sec.}$$



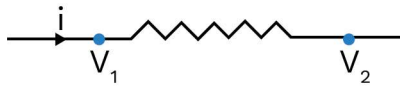
Thermal Resistance:

It is the oppose to the flow of heat through the conductor. When we compare the flow of heat to the flow of electricity.



$$i_T = \frac{dQ}{dt} = \frac{KA}{L}(T_1 - T_2)$$

(i_T = Thermal current)



$$i = \frac{V_1 - V_2}{R}$$

On comparing we get that $\frac{L}{KA}$ is the value of thermal resistance.

So $R = \frac{L}{KA}$

Combination of Rods:

1. Series combination:

In series combination heat current is same in every rod.



$$R_s = R_1 + R_2 + \dots + R_n \quad \left[\because R = \frac{L}{KA} \right]$$

$$\frac{L_1 + L_2 + L_3 + \dots + L_n}{K_s A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \dots + \frac{L_n}{K_n A}$$

$$K_s = \frac{L_1 + L_2 + L_3 + \dots + L_n}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \dots + \frac{L_n}{K_n}}$$



Concept Reminder

By using analogous behaviour between heat conduction and current conduction, we can solve heat problems as we solve electrical circuit problems.



Key Points

- ◆ Thermal resistance



Concept Reminder

Thermal resistance is inversely proportional to conductivity of heat in similar way as electrical resistance is inversely proportional to electrical conductivity.



For 2 Rods: Area and length is same

$$K_s = \frac{L + L}{\frac{L}{K_1} + \frac{L}{K_2}} = \frac{2L}{L \left(\frac{K_1 + K_2}{K_1 K_2} \right)}$$

$$K_s = \frac{2K_1 K_2}{K_1 + K_2}$$

Ex. Find the equivalent conductivity (K_{eq})



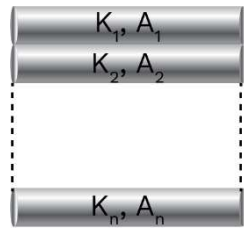
Sol. $K_{eq} = \frac{L_{eq}}{\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + \frac{\ell_3}{K_3}}$

$$\frac{8\ell}{\frac{\ell}{K} + \frac{4\ell}{2K} + \frac{3\ell}{K}} \Rightarrow \frac{16K}{2 + 4 + 6} = \frac{4}{3}K$$

2. Parallel combination:

In parallel combination temperature difference between every rod is same.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \dots \frac{1}{R_n}, \quad \left[\frac{1}{R_p} = \frac{KA}{L} \right]$$



$$\frac{K_p (A_1 + A_2 + \dots \dots + A_n)}{L} = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L} + \dots \dots \frac{K_n A_n}{L}$$

For 2 rod: $A_1 = A_2 = A$

$$K_p = \frac{K_1 A + K_2 A}{A + A} = \frac{A(K_1 + K_2)}{2A} = \frac{K_1 + K_2}{2}$$

$$K_p = \frac{K_1 + K_2}{2}$$

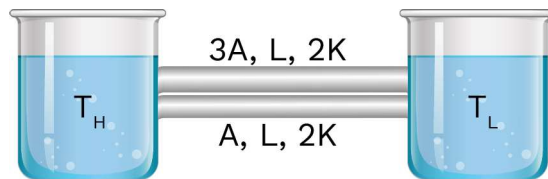
Rack your Brain



Consider a compound slab consisting of two different materials having equal thicknesses and thermal conductivities K and $2K$, respectively. Find the equivalent thermal conductivity of the slab.



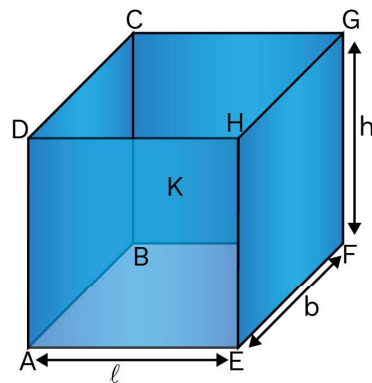
Ex. Find equivalent thermal conductivity (K_{eq})



Sol. $K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$

$$= \frac{2K(3A) + 2K(A)}{4A} \Rightarrow \frac{8KA}{4A} = 2K$$

Ex. For given rod thermal conductivity is K then find out the thermal resistance for given surface.



1. ABCD & EFGH
2. ABFE & CDHG
3. AEHD & BFGC

1. ABCD & EFGH

$$R = \frac{L}{KA} = \frac{\ell}{K(bh)}$$

2. ABFE and CDHG

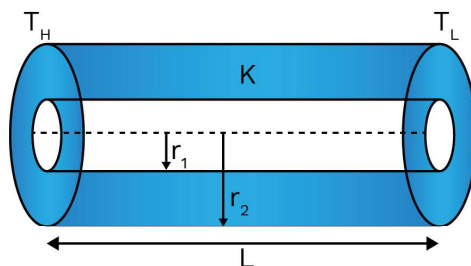
$$R = \frac{L}{KA} = \frac{h}{K(\ell \times b)}$$

3. AEHD & BFGH

$$R = \frac{L}{KA} = \frac{b}{K(\ell h)}$$



Ex. Find out thermal equivalent resistance.

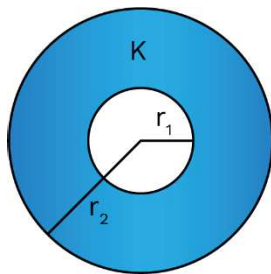


Sol. $R_{eq} = \frac{L}{KA} \Rightarrow R_{eq} = \frac{L}{K(\pi r_2^2 - \pi r_1^2)}$

$L \rightarrow$ length along heat flow direction

$A \rightarrow$ Area perpendicular to heat flow direction.

Ex. Find out thermal equivalent resistance.



Sol. $R = \frac{1}{4\pi K} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{1}{4\pi K} \left(\frac{r_2 - r_1}{r_1 r_2} \right)$

$$R \propto \frac{r_2 - r_1}{r_1 r_2}$$

$$\therefore H = \frac{\Delta T}{R} \Rightarrow H \propto \frac{1}{R}$$

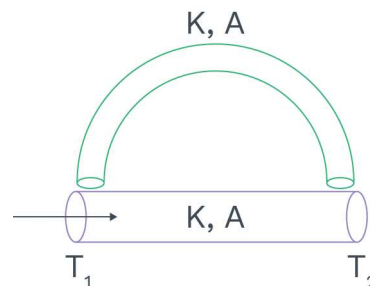
$$\therefore H \propto \frac{r_1 r_2}{r_2 - r_1}$$

Ex. Two rods are made by same material and same cross section area these are arranged as shown in figure then find out the ratio of heat current.

Rack your Brain

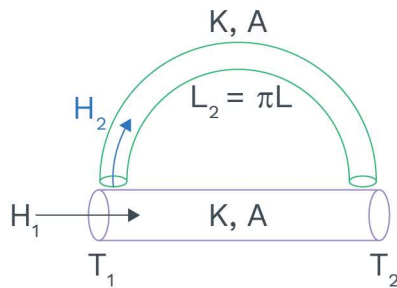


Heat is flowing through two cylindrical rods of the same material. The diameters of the rods are in the ratio 1 : 2 and the lengths in the ratio 2 : 1. If the temperature difference between the ends is same, then find out ratio of the rate of flow of heat through them.





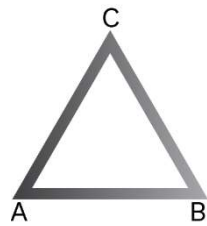
Sol.



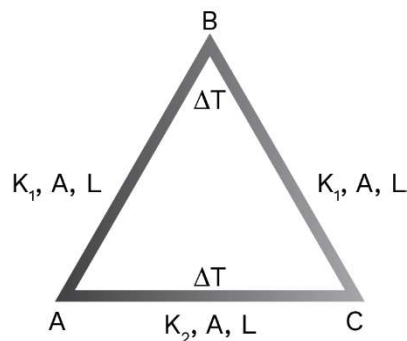
$$\therefore H = \frac{KA\Delta T}{L} \Rightarrow H \propto \frac{1}{L}$$

$$\therefore \frac{H_1}{H_2} = \frac{L_2}{L_1} = \frac{\pi L}{2L} = \frac{\pi}{2}$$

Ex. Three rods of same length and same cross section area have been arranged in an equilateral triangle A, B and C shown in figure.



Thermal conductivity of AB and BC rod is K_1 and thermal conductivity of AC rod is K_2 . If heat current in ABC and AC rod is same, then find out the relation between K_1 and K_2 .



Sol. $H_{ABC} = H_{AC}$

$$\therefore H = \frac{KA\Delta T}{L}$$

$$\Rightarrow \frac{K_1 A \Delta T}{2L} = \frac{K_2 A \Delta T}{L} \Rightarrow K_1 = 2K_2$$



Ex. In two same dimensional vessels same amount and same temperature ice is filled. If in both vessels complete ice is melt in 20 min and 50 min then find out the ratio of thermal conductivity of both vessels.

Sol. $H = \frac{Q}{t}$, $\left[\because H = \frac{KA\Delta T}{L} \right]$

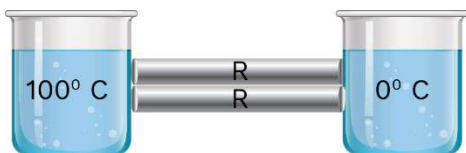
So, $H \propto \frac{1}{t}$ and $H \propto K$

$$\frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{50}{20} = \frac{5}{2}$$

Here, $R = \frac{L}{KA}$

Ex. Two identical rods are connected in series between 100° and 0°C temperature then 4 cal heat is transferred in 2 sec. If these rods are connected in parallel between same temperature difference, then find out the time in which same amount of heat is transferred.

Sol.



$$H = \frac{Q}{t} \text{ and } H = \frac{\Delta T}{R}$$

$$\Rightarrow H \propto \frac{1}{t} \text{ and } \Rightarrow H \propto \frac{1}{R}$$

So, $t \propto R$

$$\frac{t_s}{t_p} = \frac{R_s}{R_p} \Rightarrow \frac{2}{t_p} = \frac{2R}{R/2} \Rightarrow t_p = \frac{1}{2} \text{ sec.}$$



Concept Reminder

In series connection time in which heat is transferred is more as compared to time in parallel connection.

Rack your Brain



The two ends of a metal rod are maintained at temperatures 100°C and 110°C . The rate of heat flow in the rod is found to be 4.0 J/s . If the ends are maintained at temperatures 200°C and 210°C , then calculate rate of heat flow.



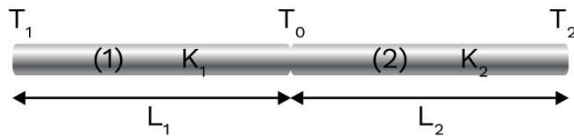
Ex. Two identical rods are connected in series then 40 cal heat is transferred. If these rods are connected in parallel between same temperature difference, then find out the heat transfer in same time interval.

Sol. $H = \frac{\Delta T}{R} \Rightarrow H \propto \frac{1}{R}$

$$\frac{H_1}{H_2} = \frac{R/2}{2R} \Rightarrow \frac{40}{H_2} = \frac{1}{4}$$

$$H_2 = 160 \text{ cal}$$

Temperature of junction/Interface Temperature (T_0):



When we have to calculate the temperature of junction then we equate the rate of heat flow from both parts.

$$\left(\frac{dQ}{dt} \right)_1 = \left(\frac{dQ}{dt} \right)_2$$

$$\frac{K_1 A}{L_1} (T_1 - T_0) = \frac{K_2 A}{L_2} (T_0 - T_2)$$

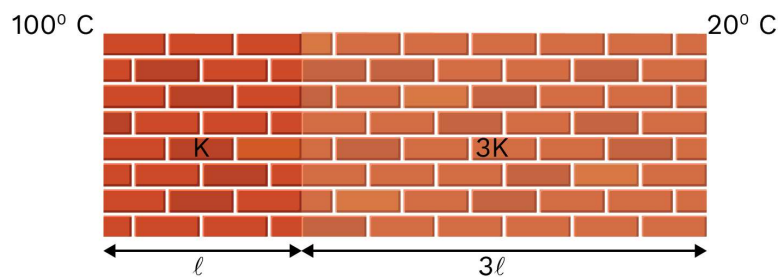
And calculate the T_0 .



Concept Reminder

When we calculate temperature of junction, we always consider our system in steady state.

Ex. Find the temperature of junction of following figure. (Cross section area is same)





Sol. $\left(\frac{KA\Delta T}{\ell}\right)_1 = \left(\frac{KA\Delta T}{\ell}\right)_2$

$$\frac{KA(100 - T)}{\ell} = \frac{3KA(T - 20)}{3\ell}$$

$$100 - T = T - 20$$

$$T = 60^\circ\text{C}$$

Ex. For given combination of rod, find out the equivalent thermal conductivity.



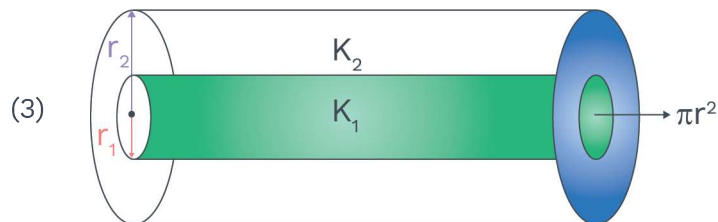
$$K_s = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}} = \frac{L + 2L}{\frac{L}{2K} + \frac{2L}{K}}$$

$$K_s = \frac{3L}{\frac{L + 4L}{2K}} = \frac{6LK}{5L} = \frac{6K}{5}$$



$$K_p = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2} = \frac{2K(\pi r^2) + K(4\pi r^2)}{\pi r^2 + 4\pi r^2}$$

$$K_p = \frac{2K + 4K}{1 + 4} = \frac{6K}{5}$$



$$K_p = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

$$A_1 = \pi r_1^2$$

$$A_2 = 4\pi r^2 - \pi r^2 = 3\pi r^2$$

$$K_p = \frac{K_1(\pi r^2) + K_2(3\pi r^2)}{\pi r^2 + 3\pi r^2} = \frac{K_1 + 3K_2}{4}$$



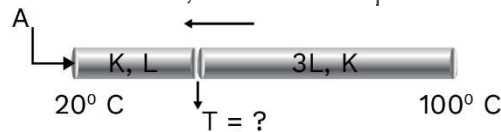
Note: If two different rods are connected in series then their thermal resistance is directly proportional to temperature difference it means ratio of resistance is equal to ratio of temperature difference.

$$H = \frac{\Delta T}{R} \text{ (For series combination H will be same)}$$

$$\Rightarrow R \propto \Delta T$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\Delta T_1}{\Delta T_2}$$

Ex. In given combination of rod, find out temperature at junction point.



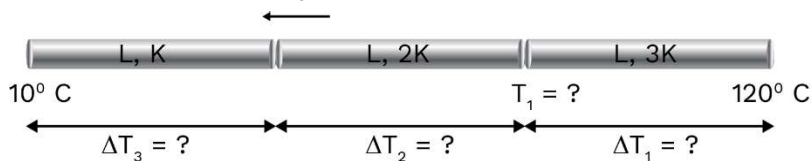
Sol. $H_1 = H_2$, $\left[\because H = \frac{KA\Delta T}{L} \right]$

$$\Rightarrow \frac{KA[100 - T]}{3L} = \frac{KA[T - 20]}{L}$$

$$\Rightarrow 100 - T = 3T - 60$$

$$\Rightarrow T = 40^\circ\text{C}$$

Ex. Find out $\Delta T_1, \Delta T_2$ and ΔT_3 in given combination.



Sol. Here given combination is a series combination.

$$\text{So, } H_1 = H_2 = H_3$$

$$\therefore H = \frac{KA\Delta T}{L}$$

$$\Rightarrow \Delta T \propto \frac{1}{K}$$

$$\text{So, } \Delta T_1 : \Delta T_2 : \Delta T_3 = \frac{1}{3K} : \frac{1}{2K} : \frac{1}{K} = 2 : 3 : 6$$

Now,

$$\Delta T_1 = \frac{2}{11} \times 110 = 20^\circ\text{C}$$

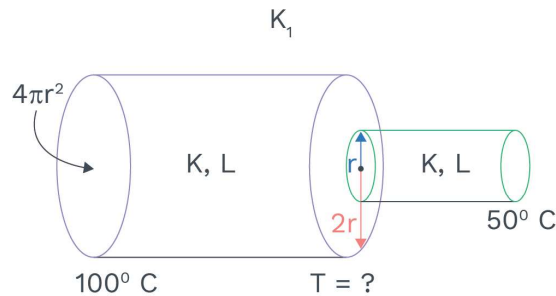
$$\Rightarrow T_1 = 120 - \Delta T_1 = 120 - 20 = 100^\circ\text{C}$$

$$\Delta T_2 = \frac{3}{11} \times 110 = 30^\circ\text{C}$$

$$\Rightarrow T_2 = T_1 - \Delta T_2 = 100 - 30 = 70^\circ\text{C}$$

$$\Delta T_3 = \frac{6}{11} \times 110 = 60^\circ\text{C}$$

Ex. Find out the temperature of junction T?



Sol. $H_1 = H_2$

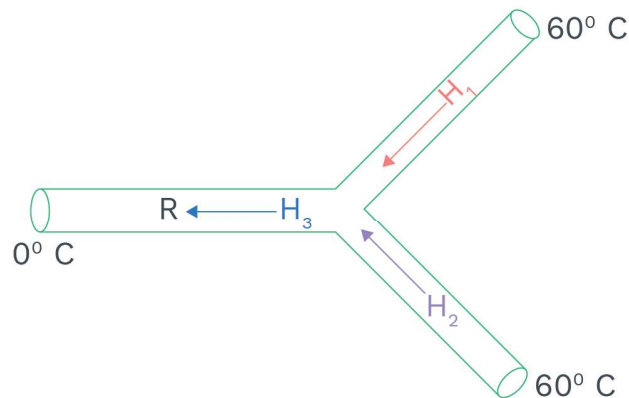
$$\left[\because H = \frac{KA\Delta T}{L} \right]$$

$$\frac{K(4\pi r^2)(100 - T)}{L} = \frac{K(\pi r^2)(T - 50^\circ)}{L}$$

$$\Rightarrow 400 - 4T = T - 50$$

$$\Rightarrow T = 90^\circ\text{C}$$

Ex. In given combination all rods are identical find out temperature at junction.



Concept Reminder

In this example, we are using fact that total amount of heat entering junction will be equal to total amount of heat leaving that junction.

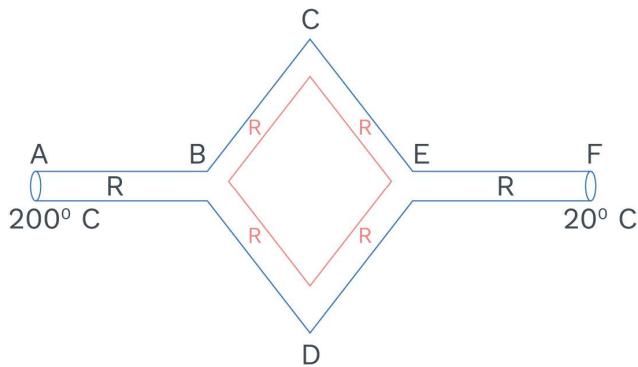
Sol. $H_1 + H_2 = H_3$, $H = \frac{\Delta T}{R}$

$$\therefore \frac{60 - T}{R} + \frac{60 - T}{R} = \frac{T - 0}{R}$$

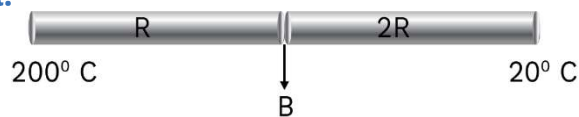
$$3T = 120^\circ \Rightarrow T = \frac{120}{3} = 40^\circ\text{C}$$



Ex. In given combination all rods are identical. Find out the temperature at junction B.



Sol.



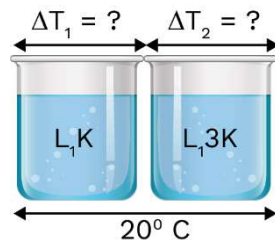
$$H_1 = H_2, \quad \therefore H = \frac{\Delta T}{R}$$

$$\frac{200 - T_B}{R} = \frac{T_B - 20}{2R}$$

$$400 - 2T_B = T_B - 20$$

$$3T_B = 420 \Rightarrow T_B = 140^\circ\text{C}$$

Ex. In given combination find out ΔT_1 and ΔT_2 ?



Sol. $H_1 = H_2 \quad \left\{ \because H = \frac{KA\Delta T}{L} \right\}$

$$\Rightarrow \Delta T \propto \frac{1}{K} \Rightarrow \Delta T_1 : \Delta T_2 :: \frac{1}{K} : \frac{1}{3K}$$

$$\Delta T_1 = \frac{3}{4} \times 20 = 15^\circ\text{C}$$

$$\Delta T_2 = \frac{1}{4} \times 20 = 5^\circ\text{C}$$

Rack your Brain

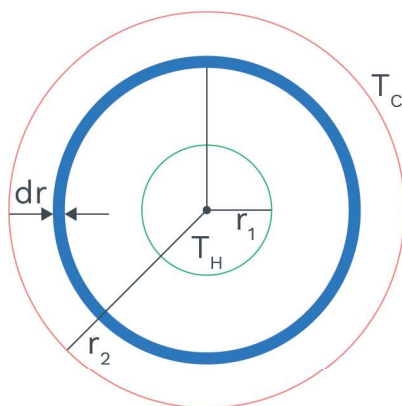


A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t . The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in thermal contact with the two reservoirs in time t .



Ex. Two thin concentric shells made of copper of radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer sphere are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres, find the value of P .

Sol.



$$dR = \frac{dr}{K \cdot 4\pi r^2} \text{ [where } R = \text{thermal resistance]}$$

$$\int_{r_1}^{r_2} dR = \int_{r_1}^{r_2} \frac{dr}{4\pi r^2 \cdot K}$$

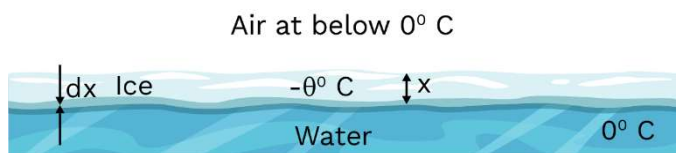
$$R = \frac{1}{4\pi K} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

thermal current

$$i_H = P = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C)(r_1 r_2)}{r_2 - r_1}$$

Growth of ice on lakes:

- In winter atmospheric temperature falls below 0°C and water in the lake start freezing. Let at time t thickness of ice on the surface of the lake = x and air temperature = $- \theta^\circ\text{C}$.
- The temperature of water in contact with the lower surface of ice = 0°C . Assume area of the lake = A



Concept Reminder

Time taken by ice to grow increases as thickness of ice increases.



Heat flow in time dt is

$$\frac{dQ}{dt} = \frac{kA(0 - (-\theta))}{x}$$

$$\frac{mL}{dt} = \frac{kA\theta}{x}, \quad \left[dQ = mL, \quad m = \int A \cdot dx \right]$$

$$\frac{\rho A dx \cdot L}{dt} = \frac{kA\theta}{x}$$

$$\int_0^t dt = \frac{\rho L}{k\theta} \cdot \int_0^x x dx$$

$$t = \frac{1}{2} \frac{\rho L}{k\theta} \cdot x^2$$

time taken by ice to grow from thickness x_1 to thickness x_2 is

$$t = t_2 - t_1 = \frac{1}{2} \frac{\rho L}{k\theta} (x_2^2 - x_1^2)$$

$$t \propto x^2$$

3. Radiation:

- The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.
- Precisely it is energy transfer in the form of electromagnetic wave through any medium.
- Heat radiation are always obtained in infra-red region of electromagnetic wave spectrum, so they are called infra-red rays.
- Medium is not required for the propagation of these radiations.
- They produce sensation of warmth on us, but we can't see them.
- Everybody whose temperature is above zero Kelvin emits thermal radiation.
- Their speed is equal to that of light in vacuum i.e. $c = 3 \times 10^8$ m/s hence it is the fastest mode of heat transfer.



Concept Reminder

Only in radiation mode of transfer, there is no requirement of medium.



Concept Reminder

Radiation is fastest mode of heat transfer. Radiation of sun take 8 min and 20 sec to reach earth.



- For point source, radiation energy obeys inverse square law i.e.
 - For point source, intensity at a point is inversely proportional to the square of the distance of point from the source.

$$I \propto \frac{1}{r^2}$$

- For line source, intensity at a point is inversely proportional to the distance of the point from the source.

$$I \propto \frac{1}{r}$$

- Just as light waves, they follow laws of reflection, refraction, interference, diffraction, and polarisation.
- When these radiations fall on a surface then exert pressure on that surface which is known as radiation pressure.
- While travelling these radiations travel just like photons of electromagnetic waves, they manifest themselves as heat only when they are absorbed by a substance.
- In order to obtain a spectrum of radiation, a special prism used like KCl prism, rock salt prism, normal glass prism or quartz prism cannot be used (because it absorbed some radiation)
- Radiation intensity measured with a specific device named as bolometer.

Basic Fundamental Definitions:

Absorptive Power or Absorptive Coefficient (a):

- The ratio of amount of radiation absorbed by a surface (Q_a) to the amount of radiation incident (Q) upon it, is defined as the coefficient of absorption

$$a = \frac{Q_a}{Q}$$

- It is unitless and dimensionless.

Spectral Absorptive Power (a_λ):

- $a_\lambda = \frac{Q_{a_\lambda}}{Q_\lambda}$; Also called monochromatic absorptive coefficient at a given wavelength.
- $a = \int_0^\infty e_\lambda \cdot d\lambda$ for an ideal black body a_λ and $a = 1$, a and a_λ are unitless.

Key Points

- Electromagnetic wave
- Radiation intensity



Rack your Brain



Find out dimensional formula for spectral emissive power.



Emissive Power (e):

- The amount of heat radiation emitted by unit surface area in unit second at a particular temperature.
- SI unit: $\text{J/m}^2\text{-s}$ or watt/m^2 .

Spectral Emissive Power (e_λ):

- The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength.
- Emissive power or total emissive power

$$e = \int_0^\infty e_\lambda \cdot d\lambda$$

SI unit: $\text{W/m}^2\text{-}\text{\AA}$

Emissivity (e):

a. Absolute emissivity or emissivity:

Radiation energy given out by a unit surface area of a body in time corresponding to unit temperature difference with respect to the surroundings is called emissivity.

SI unit: $\text{W/m}^2 \text{ K}$

b. Relative emissivity (e_r):

$$e_r = \frac{Q_{GB}}{Q_{IBB}} = \frac{e_{GB}}{E_{IBB}}$$

$$= \frac{\text{Emitted radiation by general body}}{\text{Emitted radiation by ideal black body}}$$

GB = General body, IBB = Ideal black body.

- (i) No unit.
- (ii) For ideal black body $e_r = 1$.
- (iii) For non-ideal black body $0 < e_r < 1$.
- Spectral, emissive, absorptive and transmitted power of a given body surface.

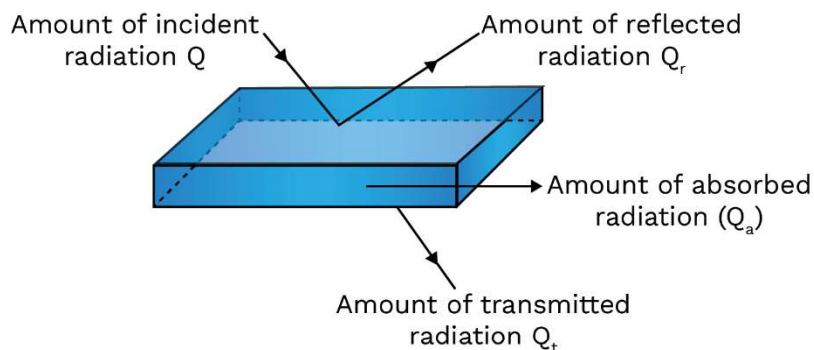
Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission.

Key Points

- ♦ Energy conservation
- ♦ Reflection
- ♦ Absorption
- ♦ Transmission



• **From energy conservation:**



$$Q = Q_r + Q_a + Q_t$$

$$\frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1$$

$$\boxed{r + a + t = 1}$$

r – Reflective coefficient, $\frac{Q_r}{Q}$

a – Absorptive coefficient, $\frac{Q_a}{Q}$

t – Transmittive coefficient, $\frac{Q_t}{Q}$

$r = 1$ and $a = 0$, $t = 0 \Rightarrow$ perfect reflector

$a = 1$ and $r = 0$, $t = 0 \Rightarrow$ ideal absorber (ideal black body)

$t = 1$ and $a = 0$, $r = 0 \Rightarrow$ perfect transmitter (diathermanous)

$$\text{Reflection power (r)} = \left[\frac{Q_r}{Q} \times 100 \right] \%$$

$$\text{Absorption power (a)} = \left[\frac{Q_a}{Q} \times 100 \right] \%$$

$$\text{Transmission power (t)} = \left[\frac{Q_t}{Q} \times 100 \right] \%$$

Ideal Black Body:

- A body surface which absorbs all incident thermal radiations at low temperature, irrespective of their wavelength and emits out all these absorbed radiations at high temperature is assumed to be an ideal black body surface.



Concept Reminder

- ♦ If a body is good absorber then it will be good emitter also.
- ♦ If a body is good reflector then it will be bad emitter.

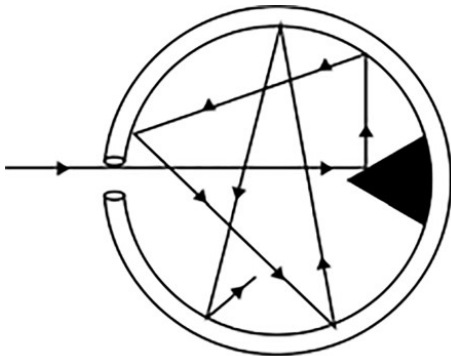
Rack your Brain



Which of the following is best close to an ideal black body?

- (1) Black lamp
- (2) Cavity maintained at constant temperature
- (3) Platinum black
- (4) A lump of charcoal heated to high temperature

- The identical parameters of an ideal black body is given by $a = a_\lambda = 1$ and $r = 0 = t$, $e_r = 1$.



Ferry's Ideal Black Body

- the nature of emitted radiations from surface of ideal black body only depends on its temperature.
- The radiation emitted from surface of ideal black body are called as either full or white radiations.
- There are two experimentally ideal black body.
- (a) Ferry's ideal black body (b) Wien's ideal black body.
- An ideal black body need not be of black colour (eg. sun)

Prevost's Theory of Heat Energy Exchange:

- According to Prevost, at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time.
- The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.
- When a cold body is placed in the hot surrounding (heating effect):** The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases. (heating effect)
- When a hot body placed in cooler surrounding (cooling effect):** The body radiates more energy and absorbs less energy from the surrounding therefore temperature of body decreases.



Concept Reminder

An ideal black body can absorb all incident electromagnetic radiation regardless of frequency and angle of incidence.



- **When the temperature of a body is equal to the temperature the surrounding:** The energy radiates per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant and the body is in thermal equilibrium with surrounding.

Kirchhoff's Law:

- At a given temperature for all bodies the ratio of their spectral emissive power (e_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at same temperature.

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant}$$

$$\left(\frac{e_\lambda}{a_\lambda}\right)_1 = \left(\frac{e_\lambda}{a_\lambda}\right)_2 = \text{constant}$$

$$e_\lambda \propto a_\lambda$$

- *Good absorbers are good emitters and bad absorbers are bad emitters.*

Stefan's Boltzmann Law:

- According to this law emissive power of an ideal black body is directly proportional to fourth power of the temperature.

$$E \propto T^4$$

$$E = \sigma T^4$$

σ = Stefan constant.

$$\sigma = 5.67 \times 10^{-8} \frac{\text{J}}{\text{m}^2 \text{ sec K}^4} \text{ or } \frac{\text{Watt}}{\text{m}^2 \text{K}^4}$$

Power

$$E = \frac{P}{A} \Rightarrow P = EA$$

For ideal black body

$$P = \sigma AT^4$$

For general body

$$\varepsilon = \frac{e}{E} \Rightarrow E = \frac{e}{\varepsilon}$$

$$P = \varepsilon \sigma AT^4$$

Heat,

$$E = \frac{Q}{AT} \Rightarrow Q = EAT$$

Key Points

- ♦ Spectral emissive power
- ♦ Ideal black body

Rack your Brain

For a black body at temperature 727°C , its radiating power is 60 watt and temperature of surrounding is 227°C . If temperature of black body is changed to 1227°C then calculate its radiating power.



For ideal black body

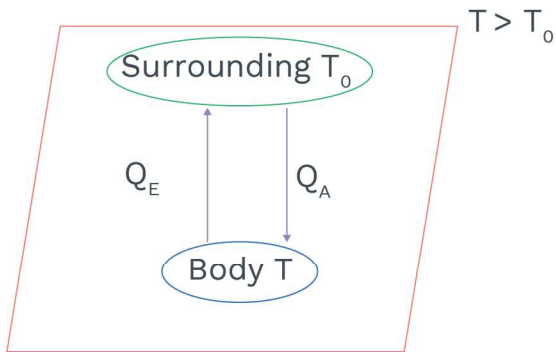
$$Q = \sigma AT^4 t$$

For general body

$$Q = \epsilon \sigma AT^4 t$$

Rate of heat loss (R_H):

- The rate of heat loss depends upon nature of surface and area of surface and temperature of body and surrounding.



Concept Reminder

Rate of heat loss R_H is $\frac{dQ}{dt}$ and
rate of cooling is $\frac{dT}{dt}$.

$$Q_{\text{loss}} = Q_E - Q_A$$

$$Q_{\text{loss}} = \epsilon \sigma AT^4 t - \epsilon \sigma AT_0^4 t$$

$$Q_{\text{loss}} = \epsilon \sigma A [T^4 - T_0^4] t$$

$$R_H = \frac{dQ_{\text{loss}}}{dt} = \epsilon \sigma A (T^4 - T_0^4)$$

T_0 = temperature of surrounding

Rate of cooling (R_c or R_θ)

$$R_\theta = -\frac{dT}{dt}$$

Rate of cooling depends upon rate of heat loss (R_θ), mass of body (m) and specific heat of body.

$$\Delta Q = mc\Delta T$$

$$\frac{dQ}{dt} = mc \frac{dT}{dt} \left(R_H = \frac{dQ}{dt} \text{ and } R_\theta = \frac{dT}{dt} \right)$$

$$R_H = mcR_\theta$$

$$R_\theta = \frac{R_H}{mc} = \frac{\epsilon \sigma A [T^4 - T_0^4]}{mc}$$



Ex. For an ideal black body, its temperature is increase by 10°C then find out the percentage change in emissive power (Take $E_i = 100 \text{ W}$ for simplicity).

Sol. $E \propto T^4$

$$\frac{E_i}{E_f} = \left(\frac{T_i}{T_f} \right)^4$$

$$\frac{100}{E_f} = \left(\frac{100}{110} \right)^4 \Rightarrow \frac{100}{E_f} = \frac{100 \times 100}{121 \times 121}$$

$$E_f = 146.41$$

Percentage change

$$E = \left(\frac{146.41 - 100}{100} \right) \times 100 = 46.41\%$$

Since temperature increases then emissive power also increases.

Ex. For 2 ideal black surface sphere radius is R and $2R$ and temperature is $2T$ and T then find out the ratio of emitted power.

Sol. $P = \sigma AT^4 \Rightarrow P \propto AT$

$$\frac{P_1}{P_2} = \frac{4\pi R_1^2}{4\pi R^2} \cdot \frac{T_1^4}{T_2^4}$$

$$\frac{P_1}{P_2} = \frac{R^2}{4R^2} \cdot \frac{(2T)^4}{T^4}$$

$$\frac{P_1}{P_2} = \frac{16}{4} = \frac{4}{1}$$

Key Points

- ◆ Emissive power
- ◆ Rate of cooling

Ex. Two same dimensional spheres are made up by same material and having same surface and having same temperature if these are placed at same temperature surrounding then compare the rate of cooling when one is solid and other is hollow.

Sol. $R_\theta = \frac{R_H}{mC} = \frac{\varepsilon\sigma A(T^4 - T_0^4)}{mC}$

$$R_\theta \propto \frac{1}{m}$$

$m = \text{solid} > \text{hollow}$

$R_\theta = \text{hollow} > \text{solid}.$



Ex. For a cube, cylinder and sphere, linear dimension is same and these are made up by same material and having same surface and same temperature it these are placed in same temperature surrounding then find out the order of rate of heat loss and rate of cooling.

Sol. $R_H = \varepsilon\sigma A[T^4 - T_0^4]$

$$R_H \propto A$$

$$\text{cube} = 6a^2$$

$$\text{cylinder} = 2\pi a^2 + \pi a^2 + \pi a^2 = 4\pi a^2$$

$$\text{sphere} = 4\pi a^2$$

$$R_H = \text{cylinder} = \text{sphere} > \text{cube}$$

$$R_\theta = \frac{R_H}{mC}$$

$$\rho = \frac{M}{V} \Rightarrow M \propto V$$

$$R_\theta \propto \frac{A}{V}$$

$$\frac{A}{V} \Rightarrow \text{cube} = \frac{6a^2}{a^3} = \frac{6}{a}$$

$$\text{cylinder} = \frac{4\pi a^2}{\pi a^3} = \frac{4}{a}$$

$$\text{sphere} = \frac{4\pi a^2}{\frac{4}{3}\pi a^3} = \frac{3}{a}$$

Rate of cooling; cube > cylinder > sphere.

Ex. For a body temperature is 600 K is placed in a surrounding of temperature 300 K then rate of cooling is R if temperature of body is 900 K, then find out the rate of cooling.

Sol. $R_\theta = \frac{\varepsilon\sigma A(T^4 - T_0^4)}{mC}$

$$R_\theta \propto T^4 - T_0^4$$

$$\frac{R}{R_\theta} = \frac{600^4 - 300^4}{900^4 - 300^4}$$

$$= \frac{100^4[6^4 - 3^4]}{100^4[9^4 - 3^4]} = \frac{3^4[2^4 - 1]}{3^4[3^4 - 1]} = \frac{15}{80}$$



Ex. Plot the curve between $\log_e E$ and $\log_e T$.

Sol. $E = \sigma T^4$

Taking $\log_e$ both side

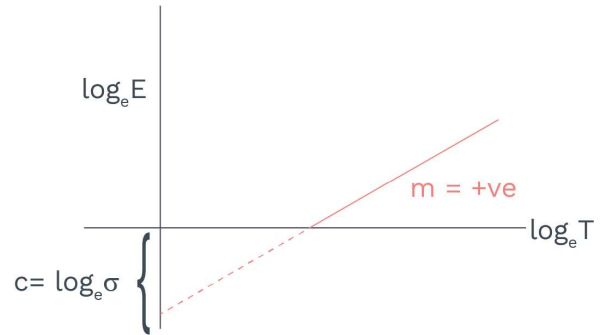
$$\log_e E = \log_e \sigma T^4$$

$$\log_e E = 4 \log_e T + \log_e \sigma$$

$$y = mx + C$$

$$\sigma = 5.67 \times 10^{-8} \quad (\sigma < 1)$$

$$\log_e \sigma = -ve \text{ (negative)}$$



Newton's Cooling Law:

- According to this law rate of cooling $\left(-\frac{dT}{dt}\right)$ is

directly proportional to excess of temperature of the body over that of surrounding.

$$-\frac{dT}{dt} \propto T - T_0 \quad (\text{when } T - T_0 \nlessgtr 35^\circ\text{C})$$

T = temperature of body [all temperature in $^\circ\text{C}$]

T_0 = temperature of surrounding

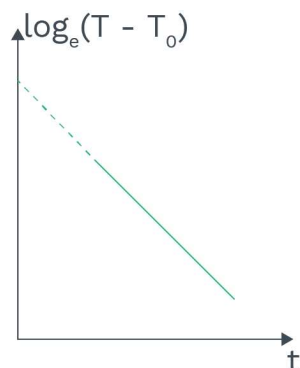
Where $-ve$ sign represent that the rate of cooling is decrease with time

$$\boxed{-\frac{dT}{dt} = K(T - T_0)}$$

- $\frac{dT}{dt} = -K(T - T_0)$

$$\int \frac{dT}{T - T_0} = \int -K dt$$

$$\log_e(T - T_0) = -Kt + C$$



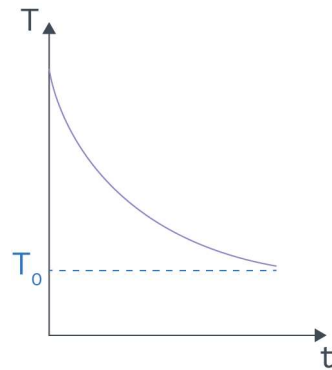
Concept Reminder

Newton's cooling law is applicable only when temperature difference between body and surrounding is not large.

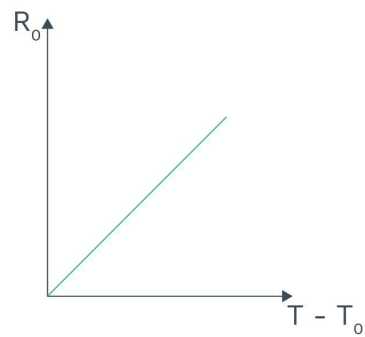
$(T - T_0)$ should not be greater than 35°C



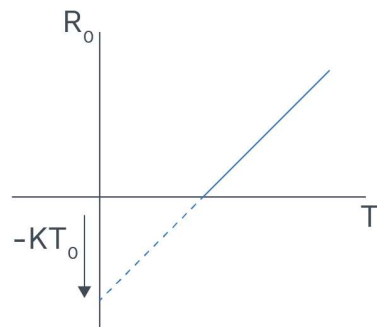
- $T - T_0 = e^{-Kt}$
 $T = T_0 + e^{-Kt}$



- $R_\theta = -\frac{dT}{dt} = K[T - T_0]$
 $T_\theta = K[T - T_0]$
 $R_\theta \propto T - T_0$



- $R_\theta = K[T - T_0]$
 $R_\theta = KT - KT_0$
 $y = mx + C$





- If temperature of body is changed $T_1 \xrightarrow{t} T_2$ in 't' time in a surrounding temperature T_0 then

$$\boxed{\frac{T_1 - T_2}{t} = K \left[\frac{T_1 + T_2}{2} - T_0 \right]}$$

Ex. If 2 spheres of equal radius have same density same specific heat and both have same temperature T and surrounding temperature is T_s .

Then comment on:

(i) Rate of heat loss (ii) Rate of cooling.

- (1) $1 > 2$ (2) $1 < 2$
 (3) $1 = 2$ (4) None of these

Sol. (i) $\frac{dQ}{dT} = R_H$ ($R_H \propto r^2$)

$$(ii) \frac{dR_{\theta_1}}{dR_{\theta_2}} = \frac{\left(\frac{\sigma A_1 (T^4 - T_s^4)}{ms} \right)}{\left(\frac{\sigma A_2 (T^4 - T_s^4)}{ms} \right)} \Rightarrow \frac{dR_{\theta_1}}{dR_{\theta_2}} = \frac{\pi r_1^2}{\pi r_2^2}$$

$$dR_{\theta_1} = dR_{\theta_2}$$

Ex. A body takes 10 min to cool from 90°C to 80°C . If the room temperature is 25°C . How much time will it take to cool from 80°C to 70°C -

Sol. $\frac{10}{10} = K \left(\frac{170}{2} - 25 \right) \dots(i)$

$\frac{10}{t} = K \left(\frac{150}{2} - 25 \right) \dots(ii)$

Equation (i) divided by (ii)

$$\frac{t}{10} = \frac{60}{50} \Rightarrow t = 12 \text{ min}$$

Ex. For a body temperature is 80°C is placed in a surrounding temperature 20°C then rate of heat loss is 60 cal/sec then find out the rate of heat loss when its temperature is 40°C .

Sol. $R_\theta = \frac{R_H}{mC} \Rightarrow R_\theta \propto R_H \propto T - T_0$

$$\frac{60}{R'_H} = \frac{60}{20}$$

$$R'_H = 20 \text{ cal / sec}, \{T - T_0 \neq 35^\circ\text{C},$$

$$80^\circ\text{C} - 20^\circ\text{C} = 60^\circ\text{C}, 40^\circ\text{C} - 20^\circ\text{C} = 20^\circ\text{C}\}$$



Ex. A body temperature is change 90°C to 60°C in 5 minutes then find out the taken time when its is change 60°C to 30°C when temperature of surrounding is 20°C.

Sol.
$$\frac{T_1 - T_2}{t} = K \left[\frac{T_1 + T_2}{2} - T_0 \right]$$

$$\frac{90 - 60}{5} = K \left[\frac{90 + 60}{2} - 20 \right]$$

$$\frac{60 - 30}{t} = K \left[\frac{60 + 30}{2} - 20 \right]$$

$$\frac{t}{5} = \frac{55}{25} \Rightarrow t = 11 \text{ min}$$

Limitations of Newton's law of cooling:

- Temperature difference should not exceed 35°C.
($T - T_0$) $\nless 35^\circ\text{C}$
- Newton's law of cooling is used to calculate the specific heat of liquid because loss of heat should only be by radiation.
- This law is an extended form of Stefan-Boltzmann's law.
- Derivation of Newton's law from Stefan's Boltzmann law.

$$\frac{dT}{dt} = \frac{\sigma A}{ms} (T^4 - T_0^4) \quad (T - T_0 = \Delta T, \quad T = T_0 + \Delta T)$$

$$\frac{dT}{dt} = \frac{\sigma A}{ms} [(T_0 + \Delta T)^4 - T_0^4]$$

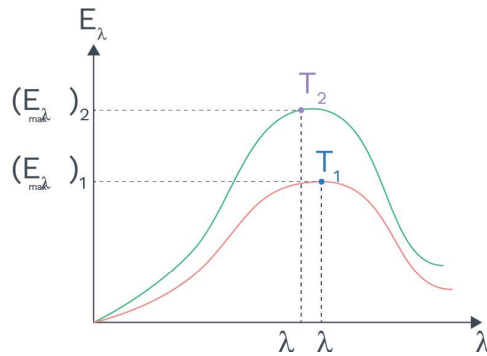
If $x \ll 1$ then $(1 + x)^n = 1 + nx$ (binomial theorem)

$$\begin{aligned} \frac{dT}{dt} &= \frac{\sigma A}{ms} \left[T_0^4 \left(1 + \frac{\Delta T}{T_0} \right)^4 - T_0^4 \right] \\ &= \frac{\sigma A}{ms} T_0^4 \left[\left(1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right] \\ &= \frac{\sigma A}{ms} T_0^4 \left[1 + 4 \cdot \frac{\Delta T}{T_0} - 1 \right] = \left(4 \frac{\sigma A}{ms} T_0^3 \right) \Delta T \end{aligned}$$

$$\boxed{\frac{dT}{dt} = K \cdot \Delta T}, \quad K = \frac{4 \sigma A T_0^3}{ms} = \text{constant}$$

Newton's law of cooling

$$\boxed{\frac{dT}{dt} \propto \Delta T} \text{ for small temperature difference.}$$

**Wien's Law:**

$$T_2 > T_1$$

$$\lambda_{m_1} > \lambda_{m_2}$$

Statement: The wavelength corresponding to maximum emission of radiation decrease with increasing temperature $\left[\lambda_m \propto \frac{1}{T} \right]$. This is known as Wien's displacement law.

$$(E_{\lambda_{\max}})_2 > (E_{\lambda_{\max}})_1$$

$$T_2 > T_1 \text{ and } \lambda_{m_1} > \lambda_{m_2}$$

$$T \propto \frac{1}{\lambda_m} \Rightarrow \boxed{T = \frac{b}{\lambda_m}} = \text{Wien's law; } b = \text{Wien's}$$

constant ($2.91 \times 10^{-3} \text{ MK}$)

y v/s x curve

$$\text{Area} = \int g \cdot dx$$

E_λ v / s λ curve

$$\text{Area} = \int E_\lambda d\lambda$$

$$\text{Area} = E$$

$$\text{Area} \propto T^4$$

$$\boxed{E_\lambda \propto T^4}$$

Relation between frequency and temperature

$$v_m = \frac{c}{b} - T \quad (c = \lambda \times v)$$

- For ideal black body spectral energy curve always be continuous.
Dimension of b is $[L\theta]$

**Concept Reminder**

- Change in colour of rod while heating can be explained using Wien's displacement law.
- At low temperature, wavelength is higher and at high temperature, wavelength is lower.

Rack your Brain

The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0 . If the temperature of the black body is now changed so that it radiates maximum energy at wavelength $\frac{3}{4}\lambda_0$, the power radiated by it becomes nP. Find out the value of n.



Ex. If at 227°C temperature wavelength is $3000\text{\AA}$ then find out the temperature at which wavelength is $6000\text{\AA}$.

Sol. $T \propto \frac{1}{\lambda_m}$

$$\frac{T_i}{T_f} = \frac{\lambda_f}{\lambda_i} \Rightarrow \frac{500}{T_f} = \frac{6000}{3000}$$

$\Rightarrow T_f = 250 \text{ K}$

Ex. For an ideal black body wavelength is change from λ to $\frac{3}{4}\lambda$ then find out the emissive power when initial emissive power is E_0 .

Sol. $E \propto T^4$

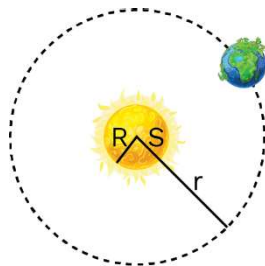
$$\frac{E_i}{E_f} = \left(\frac{T_i}{T_f}\right)^4 \Rightarrow \frac{E_0}{E_f} = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

$$E_f = \frac{256}{81}E_0$$

$$T \propto \frac{1}{\lambda_m}$$

$$\frac{T_i}{T_f} = \frac{\lambda_f}{\lambda_i} = \frac{3/4\lambda}{\lambda} \Rightarrow \frac{T_i}{T_f} = \frac{3}{4}$$

Solar Constant (S):



- The power exert by unit area of any planet is K/a solar constant.
Sun is an ideal black body so

$$E = \sigma T^4$$

$$\sigma = 5.7 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

$$1\text{m}^3 \rightarrow P = \sigma T^4$$

$$4\pi R^2 \rightarrow (\sigma T^4)4\pi R^2$$

$$\text{Total power} = (\sigma T^4)4\pi R^2$$

$r \rightarrow$ distance between sun and planet surface.
Planet



Concept Reminder

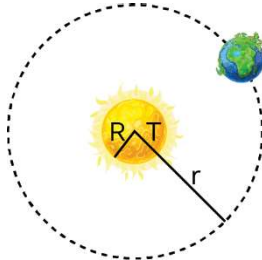
- Solar constant can be used to find out the temperature of sun.

$$4\pi r^2 \rightarrow (\sigma T^4)4\pi R^2$$

$$1 \text{ m}^2 \rightarrow \frac{(\sigma T^4)4\pi R^2}{4\pi r^2}$$

$$\boxed{S = \frac{\sigma T^4 R^2}{r^2}} \quad \boxed{S \propto \frac{1}{r^2}}$$

Ex. Assuming radius of sun is R and temperature is T and a planet radius is R_0 is placed r distance from sun then find out the total power absorbed by the planet.



Sol. Power absorbed by unit area of a any surface at r distance from sun

$$S = \frac{\sigma T^4 R^2}{r^2}$$

Total power absorbed by planet at r from sun

$$\pi(R_0^2) = P_{\text{total}} = \left(\frac{\sigma T^4 R^2}{r^2} \right) \pi R_0^2$$

Wiendmann Franz Law:

$$\frac{K}{\sigma T} = \text{constant}, \quad K \propto \sigma$$

- According to this law at given temperature T ratio of thermal conductivity and electric conductivity is constant.
- A substance is which is good conductor of heat is also a good conductor of electricity and vice versa.
- Mica $\rightarrow$ It is a good conductor of heat but bad conductor of electricity.



EXAMPLE

Q1 The temperature of three isothermal surfaces situated at right angles to the direction of heat-flow are respectively 253°C, 248°C and 243°C. Their distances from the heat-source are respectively 10 cm, 14 cm and 16 cm. Find out temperature-gradients between first and second surfaces, and between second and third surfaces-

Sol. Distance between first and second surfaces is

$$\Delta x = 14 - 10 = 4 \text{ cm} = 0.04 \text{ m}$$

Temperature difference,

$$\Delta \theta = 248 - 253 = -5^\circ\text{C}$$

$\therefore$ temperature gradient,

$$\frac{\Delta \theta}{\Delta x} = \frac{-5}{0.04} = -125^\circ\text{C} / \text{m}$$

Similarly, between second and third surfaces

$$\Delta x = 16 - 14 = 2 \text{ cm} = 0.02 \text{ m}$$

and $\Delta \theta = 243 - 248 = -5^\circ\text{C}$

$\therefore$ temperature gradient,

$$\frac{\Delta \theta}{\Delta x} = \frac{-5}{0.02} = -250^\circ\text{C} / \text{m}$$

Q2 One face of an aluminium cube of edge 2 metre is maintained at 100°C and the other end is maintained at 0°C. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (thermal conductivity of aluminium is 209 W/m-°C)

Sol. Heat will flow from the end at 100°C to the end at 0°C.

Area of cross-section perpendicular to direction of heat flow,

$$A = 4\text{m}^2$$

$$\Rightarrow \frac{Q}{t} = KA \left(\frac{T_H - T_C}{L} \right)$$

$$Q = \frac{(209 \text{ W} / \text{m}^\circ\text{C})(4\text{m}^2)(100^\circ\text{C} - 0^\circ\text{C})(5 \text{ sec})}{2\text{m}} = 209 \text{ KJ}$$



Q3 A compound bar 2 m long is constructed of a solid copper core 1cm is diameter surrounded by a steel casing whose outer diameter is 2 cm. The outer surface of the bar is thermally insulated and one is maintained at 100°C, the other is at 0°C.

- (i) Find the total heat current in the bar
 (ii) What percentage is carried by each material
 K for copper = 0.9 cal-cm⁻¹ sec⁻¹ (°C)⁻¹
 and K for steel = 0.12 cal-m⁻¹ sec⁻¹ (°C)⁻¹

Sol. For the compound bar, the rate of flow through copper bar is given by

$$\frac{Q_1}{t} = \frac{K_1 \pi r_1^2 (T_1 - T_2)}{d}$$

where r_1 is the radius

$$\text{so, } \frac{Q_1}{t} = \frac{9.1 \times 3.14 \times (0.5)^2 \times 100}{200}$$

$$\text{for steel bar } \frac{Q_2}{t} = \frac{0.12 \left[3.14 \left\{ (1)^2 - (0.5)^2 \right\} \right] \times 100}{200}$$

$$\begin{aligned} \text{(i) Total heat current in the bar } \frac{Q}{t} &= \frac{Q_1 + Q_2}{t} \\ &= \frac{9.0 \times 3.14 \times (0.5)^2 \times 100}{200} + \frac{0.12 \times 3.14 \times 0.75 \times 100}{200} \\ &= \frac{3.14 \times 100}{200} [9.0 \times 0.25 + 0.12 \times 0.75] \\ &= \frac{3.14 \times 100}{200} \times [0.225 + 0.09] \\ &= \frac{3.14 \times 0.315}{2} \end{aligned}$$

= 0.4945 cal per second.

(ii) On simplification, we get

$$\begin{aligned} \frac{Q_1}{Q} &= \frac{0.9(0.5)^2}{0.9 \times (0.5)^2 + 0.12 \times 0.75} \\ &= \frac{0.9 \times 0.25}{0.315} = \frac{0.9}{1.26} = 71.42\% \end{aligned}$$

$$\frac{Q_2}{Q} = 100 - 71.42 = 28.58\%$$



Q4 The heat is flowing through a rod of length 50 cm and area of cross-section 5 cm². Its ends are respectively at 25°C and 125°C. The coefficient of thermal conductivity of the material of the rod is 0.092 kcal/m × s × °C. Find temperature gradient in the rod.

Sol. Temperature gradient = $\frac{\Delta\theta}{\Delta x} = \frac{\theta_2 - \theta_1}{\Delta x} = \frac{125 - 25}{50} = 2^\circ\text{C} / \text{cm}$.

Q5 The length of a rod of aluminium is 1.0 meter and its area of cross-section is 5.0 cm². Its one end is kept at 250°C and the other end at 50°C. How much heat will flow in the rod in 5.0 minutes? K for Al = 2.0 × 10⁻¹ kilo-joule/(meter-second-°C). Mention the required conditions.

Sol. Length of the aluminium rod, $l = 1.0$ m, temperature – difference $\theta_1 - \theta_2 = 250 - 50 = 200^\circ\text{C}$, area of cross-section of the rod, $A = 5.0 \text{ cm}^2 = 5.0 \times 10^{-4} \text{ m}^2$,
 $K = 2.0 \times 10^{-1} \text{ kJ}/(\text{m-s-}^\circ\text{C}) = 200 \text{ J}/(\text{m-s-}^\circ\text{C})$ and time $t = 5.0$ minutes = $5.0 \times 60 = 300$ s. Hence the quantity of heat flowing in 5 minutes is given by

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

$$= \frac{200 \times (5.0 \times 10^{-4}) \times 200 \times 300}{1.0} = 6000 \text{ J}$$

$$= \frac{6000}{4.18} = 1435.5 \text{ cal} \quad [\because 1 \text{ cal} = 4.18 \text{ J}]$$

Q6 Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined. The points A and B are maintained at different temperature. What is the ratio of the heat transferred through a cross-section of a semi-circular rod to the heat transferred through a cross-section of the straight rod in a given time?



Sol. $\frac{dQ}{dt} = \frac{KA\Delta\theta}{l}$, For both rods K, A and $\Delta\theta$ are same

$$\therefore \frac{dQ}{dt} \propto \frac{1}{l}$$

$$\text{So } \frac{(dQ/dt)_{\text{semicircular}}}{(dQ/dt)_{\text{straight}}} = \frac{l_{\text{straight}}}{l_{\text{semicircular}}} = \frac{2r}{\pi r} = \frac{2}{\pi}.$$

Q7 There are three thermometers – one in contact with the skin of the man, other in between the vest and the shirt and third in between the shirt and coat. The readings of the thermometers are 30°C, 25°C and 22°C respectively. If the vest and shirt are of the same thickness, find the ratio of their thermal conductivities.

Sol. Rate of flow of heat will be equal in both vest and shirt

$$\therefore \frac{K_{\text{vest}} A_{\text{vest}} \Delta\theta_{\text{vest}} t}{l} = \frac{K_{\text{shirt}} A_{\text{shirt}} \Delta\theta_{\text{shirt}} t}{l}$$

$$\Rightarrow \frac{K_{\text{vest}}}{K_{\text{shirt}}} = \frac{\Delta\theta_{\text{shirt}}}{\Delta\theta_{\text{vest}}}$$

$$\Rightarrow \frac{K_{\text{vest}}}{K_{\text{shirt}}} = \frac{25 - 22}{30 - 25} = \frac{3}{5}.$$

Q8 The coefficients of thermal conductivity of copper, mercury and glass are respectively K_c , K_m and K_g such that $K_c > K_m > K_g$. If the same quantity of heat is to flow per second per unit area of each and corresponding temperature gradients are respectively X_c , X_m and X_g . Relate temperature gradients.

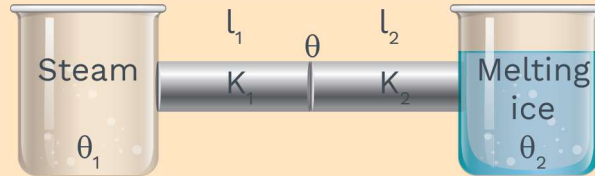
Sol. $\frac{dQ/dt}{A} = K \left(\frac{\Delta\theta}{\Delta x} \right) \Rightarrow$ Rate of flow of heat per unit area = Thermal conductivity' Temperature gradient

$$\text{Temperature gradient } \frac{d\theta}{dx} \propto \frac{1}{\text{Thermal conductivity (K)}} \quad \left[\text{As } \frac{dQ/dt}{A} = \text{constant} \right]$$

As $K_c > K_m > K_g$ therefore $X_c < X_m < X_g$.



Q9 Two rods of equal area of cross-section and length in ratio 1 : 2 and coefficient of thermal conductivity in ratio 2 : 1 are joined end to end and logged between steam and melting ice.



Find the

- (i) ratio of the temperature difference across the two rods.
- (ii) temperature of common point in the rods.

Sol. (i) $\frac{\theta_1 - \theta}{l_1 / K_1} = \frac{\theta - \theta_2}{l_2 / K_2} \Rightarrow \frac{\theta_1 - \theta}{\theta - \theta_2} = \frac{l_1}{K_1} \times \frac{K_2}{l_2} = \frac{1}{2K_1} \times \frac{K_1}{2l} = \frac{1}{4}$

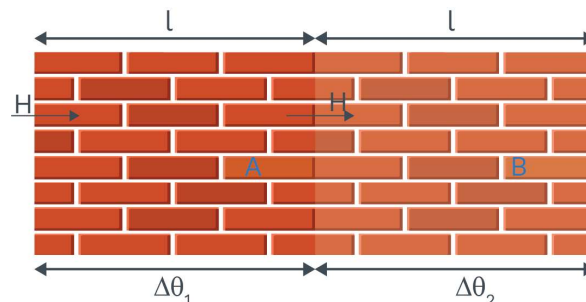
Hence ratio of temperature difference = 1 : 4

(ii) As $\frac{\theta_1 - \theta}{\theta - \theta_2} = \frac{1}{4} \Rightarrow 4(100 - \theta) = \theta - 0$

$\therefore \theta = 80^\circ\text{C}$

Q10 A wall has two layers A and B, each made of a different material, both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium, the temperature difference across the wall is 36°C . The temperature difference across the layer A.

Sol. Let $\Delta\theta_1$ and $\Delta\theta_2$ be the temperature differences across layers A and B.



In series rate of flow of heat is same $H_1 = H_2$

$$\frac{K_1(\Delta\theta_1)}{l} = \frac{K_2(\Delta\theta_2)}{l} \quad \text{i.e.} \quad \frac{\Delta\theta_1}{\Delta\theta_2} = \frac{K_2}{K_1}$$

Given $K_1 = 2K_2 \therefore \frac{\Delta\theta_1}{\Delta\theta_2} = \frac{K_2}{K_1} = \frac{1}{2} \Rightarrow \Delta\theta_2 = 2\Delta\theta_1$

Also in series $\Delta\theta_1 + \Delta\theta_2 = 36^\circ$

$$\Delta\theta_1 + 2\Delta\theta_1 = 36^\circ$$

$$3\Delta\theta_1 = 36^\circ$$

$$\Delta\theta_1 = \frac{36}{3} = 12^\circ\text{C}$$

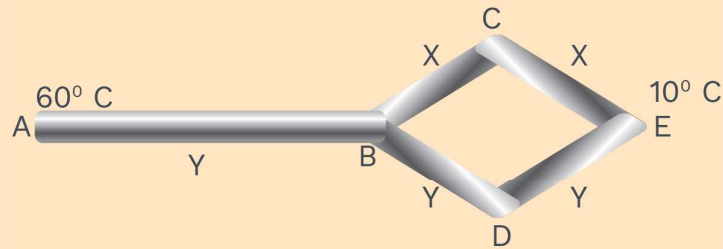
Q11 Two rods of equal length and area of cross section are kept parallel and logged between the temperature θ_1 and θ_2 . The ratio of their thermal conductivity is 2 : 3. Find the ratio of effective thermal conductivity (i) to that of first rod and (ii) to that of second rod.

Sol. $K = \frac{K_1 + K_2}{2} = \frac{2K_1 + 3K_1}{2} = \frac{5K_1}{2}$

(i) ratio of effective coefficient of that of first rod $= K : 2K = \frac{5K_1}{2} : 2K_1 = 5 : 4$

(ii) ratio of effective coefficient to that of second rod $= K : 3K = \frac{5}{2}K_1 : 3K_1 = 5 : 6$

Q12 Three rods of material X and three rods of material Y are connected as shown in figure. All the rods are identical in length and cross sectional area. If the end A is maintained at 600°C and the junction E at 10°C , calculate the temp. of the junctions B, C, D. The thermal conductivity of X is 0.92 CGS units and that of Y is 0.46 CGS units.



Sol. $R_X \propto \frac{1}{K_X}, R_Y \propto \frac{1}{K_Y} \Rightarrow \frac{R_X}{R_Y} = \frac{K_Y}{K_X} = \frac{0.46}{0.92} = \frac{1}{2}$

Let $R_X = R \therefore R_Y = 2R$

The total resistance $\Sigma R = R_Y + \text{effective resistance in the bridge}$

Further



$$l_{BCE}(2R) = l_{BDE}(4R) \text{ and } l_{BCE} + l_{BDE} = l$$

$$l_{BCE} = \frac{2}{3}l \text{ and } l_{BDE} = \frac{1}{3}l$$

For A and B

$$\theta_A - \theta_B = 60^\circ - \theta_B \Rightarrow 60 - \theta_B = 2R \times l \dots(i)$$

For B and C

$$\theta_B - \theta_C = \frac{2}{3}(1 \times R) \dots(ii)$$

$$\theta_C - \theta_E = \frac{2}{3} \times R \times l$$

For A and E

$$\theta_A - \theta_E = 60 - 10 = 50$$

$$\frac{10}{3}(R \times l) = 50$$

$$\therefore R \times l = 15$$

$$\therefore \theta_A - \theta_B = 2 \times 15 = 30, \theta_B = 60 - 30 = 30^\circ\text{C}$$

$$\theta_B - \theta_C = \left(\frac{2}{3}\right) \times 15 = 10$$

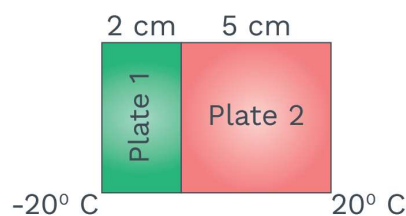
$$\therefore \theta_C = 30 - 10 = 20^\circ\text{C}$$

Obviously, $\theta_C = \theta_D = 20^\circ\text{C}$

Q13 Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is -20°C and that of the external surface of the second plate is 20°C . What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.

Sol. Rate of flow of heat in the plates is

$$\frac{Q}{t} = \frac{K_1 A (\theta_1 - \theta)}{L_1} = \frac{K_2 A (\theta - \theta_2)}{L_2} \dots(i)$$



(i) Here $\theta_1 = -20^\circ\text{C}$, $\theta_2 = -20^\circ\text{C}$, $L_1 = 2\text{ cm} = 0.02\text{ m}$, $L_2 = 5\text{ cm} = 0.05\text{ m}$ and

$$K_1 = K_2 = K$$

$\therefore$ Equation (i) becomes

$$\frac{KA(-20 - \theta)}{0.02} = \frac{KA(\theta - 20)}{0.05}$$

$$\therefore 5(-20 - \theta) = 2(\theta - 20)$$

$$\Rightarrow -100 - 5\theta = 2\theta - 40$$

$$\Rightarrow 7\theta = -60 \Rightarrow \theta = -8.6^\circ\text{C}$$

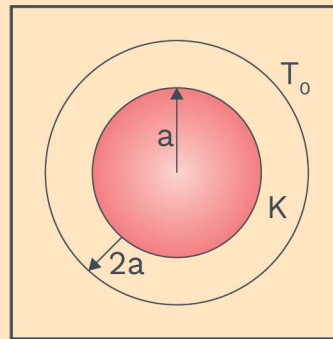
$$(ii) \frac{K_1}{K_2} = \frac{2}{5} \text{ or } K_1 = \frac{2}{5}K_2$$

$\therefore$ from equation (i)

$$\frac{\frac{2}{5}K_2A(-20 - \theta)}{0.02} = \frac{K_2A(\theta - 20)}{0.05} \Rightarrow -20 - \theta = \theta - 20$$

$$\Rightarrow -2\theta = 0 \text{ so, } \theta = 0^\circ\text{C}$$

Q14 A hollow spherical ball of inner radius a and outer radius $2a$ is made of a uniform material of constant thermal conductivity K . The temperature within the ball is maintained at $2T_0$ and outside the ball it is T_0 . Find, (a) the rate at which heat flows out of the ball in the steady state, (b) the temperature at $r = 3a/2$, where r is radial distance from the centre of shell. Assume steady state condition.



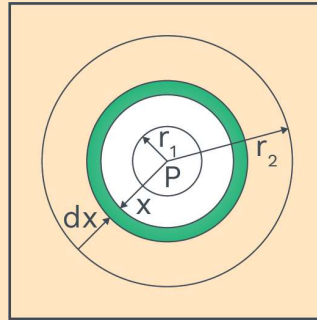
Sol. In the steady state, the net outer thermal current is constant, and does not depend on the radial position. Thermal current,

$$C_1 = \left(\frac{dQ}{dt} \right) = K \cdot (4\pi r^2) \frac{dT}{dr} \Rightarrow \frac{dT}{dr} = -\frac{C_1}{4\pi K r^2}$$

$$T = \frac{C_1}{4\pi K r} + C_2$$



Q15 Two thin concentric shells made of copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P .



Sol. Heat flowing per second through each cross-section of the sphere = $P = i$.
Thermal resistance of the spherical shell of radius x and thickness dx ,

$$dR = \frac{dx}{K \cdot 4\pi x^2}$$

$$\Rightarrow R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2 \cdot K} = \frac{1}{4\pi K} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

thermal current

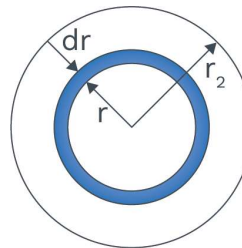
$$i = P = \frac{T_H - T_C}{R} = \frac{4\pi K (T_H - T_C) r_1 r_2}{(r_2 - r_1)}$$

Q16 Calculate thermal conductance for radial flow of a spherical shell of inner and outer radius r_1 and r_2 . Assume that thermal conductivity of the material is K

Sol.

$$\int dR = \int_{r_1}^{r_2} \frac{dr}{4\pi r^2 k}$$

$$R = \frac{(r_2 - r_1)}{4\pi r_1 r_2 k}, \quad G = \frac{1}{R} = \frac{4\pi r_1 r_2 k}{(r_2 - r_1)}$$





Mind Map

